ACCOUNTING IN A GOLDEN RULE ECONOMY

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WP 982-78 March 1978

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Presented at the American Accounting Association's 1978 North East Regional Meeting, March 16-18, 1978.

Precis

One of the commonest ratios used by investors, analysts, and corporate management to evaluate a company's or division's performance, is the return on investment, or net income produced to net book value of assets employed.

A number of writers have pointed out that this ratio is potentially extremely misleading. It is used, apparently, as a surrogate for an in-internal rate of return, but is affected by the deprecation method used and the life of the asset. More generally, the ratio is affected by the rules used for asset valuation over time.

This article demonstrates that all those difficulties disappear in an economy growing at the biological rate and argues that, within the assumptions usually used to criticize the use of the ROI, the biological rate is the economic equilibrium rate, i.e., the result is not a <u>curiosum</u>, but rather an equilibrium result.

The article concludes with the argument that accountants should attempt to make more use of the equilibrium method which has proved over the decades to be such a powerful tool in the economist's arsenal.

[redacted]

Introduction: Investment theory suggests that one should proceed by first estimating a future stream of benefits or quasi-rents and then discounting those at an appropriate rate of interest usually termed the cost of capital. The resulting present value should be compared with the cost of the investment to yield a net present value. If this be positive, one is advised, *ceteris paribus*, to proceed.

An alternative method, which is unambiguous in accept-reject decisions involving only positive quasi-rents, is to calculate an internal rate of return. This is, of course, the effective rate being earned on the cost of the investment and on the cash throwoff over the life of the investment. If it is greater than the rate of interest used above, and it will be if the net present value is positive, one is again advised, *ceteris paribus*, to proceed.

By contrast with the approach of investment theory, investors and analysts typically evaluate a company's performance by calculating the ratio of net income to total assets or variations on this such as net income plus interest expense to total assets or net income to equity. These ratios, known variously as return on investment, or accounting rates of return, are also used by corporate management to evaluate the performance of a division.

Implicit in the use of these ratios appears to be an assumption that they reflect the economic well being (and prospects?) of the company or division. More specifically, they seem to be used as a surrogate for the internal rate of return or effective rate of profit. This assumption has come under fire from a number of authors who have shown that the ROI is affected by the depreciation method employed as well as the assumed life of the asset and the growth rate of the company. Its use is, therefore, potentially misleading.

This paper begins by demonstrating these points. Some results of a deterministic simulation of the ROI models are then presented to illustrate the nature and size of the bias introduced. This is followed by an argument that given the assumptions used, one can expect the growth rate to be the biological growth rate, i.e., for the Golden Rule to apply. The article concludes by demonstrating that in a Golden Rule economy the ROI is equal to the internal rate of return and is entirely independent of the life of the asset or indeed of any accounting methods.

Accounting Rates - in Equilibrium

The study of the relationship between accounting rates of return and internal rates of return dates back at least to Solomon's (1956) seminal work. A number of fairly specific and generally unrelated results have been derived for the discrete case. The propositions offered here are, as far as I know, original and of greater generality than anything in the literature.[1]

Several caveats are in order before proceeding. All the analysis takes place in a world of certainty. Alternatively it is a world where all expectations are fulfilled. This is plainly a severe restriction and later work must attempt to relax this assumption. Taxes insofar as they are not part of normal costs are likewise excluded. In a riskless world this is probably of lesser import. However, following Miller & Modigliani (1958), the potential risk differential that taxes introduce in an uncertain world can confidently be expected to affect the results presented here. In line with this lack of uncertainty no distinction is drawn between bond holders and common stock holders. There are, of course, no changes in technology, or in relative prices, or in the general price level. The excuse must be that Rome was not built in a day.

The analysis begins with the relationship between the accounting rate of return and the internal rate of return in a stationary state as defined in the last chapter. Since this is a state which repeats itself period after period unchanged it seems to make sense to assume a constant quasi-rent stream i.e., $q_s = q$ for all s and to assume that the firm possesses a balanced stock of assets. Many of the variations that are played on this theme such as fluctuating quasi-rent patterns are reminiscent of the false problems that are created by assuming the existence of a separate class of commonstock holders in a riskless world. In other words, the first proposition is probably as general as one would ever want to go in a stationary, no growth analysis.

Assume that the firm purchases assets at the start of a year at a cost p_{ot}, or more briefly p_o, since this is assumed to be constant. Assume further that these assets generate a stream of quasi-rents q such that at the market rate of interest , assumed constant, and equal incidentally to the internal rate, we have

$$p_o = \sum_{s=1}^{n} q\,(1+\rho)^{-s} \qquad \ldots(1)$$

where n is the assumed life of the asset. For notational simplicity write

$$(1+\rho)^{-1} = v \qquad \ldots(2)$$

so that $$p_o = q\sum_{s=1}^{n} v^s = \frac{q\,(1-v^n)}{\rho} \qquad \ldots(3)$$

Note further in passing and for later reference that

$$\frac{1-v}{v} = \rho \qquad \ldots(4)$$

Now assume that the firm depreciates or more correctly allocates the cost of the asset over the n periods in such a way that the book value

at any point of time of a single asset is

$$B_t = w_t p_o \qquad \ldots(5)$$

where w_o is clearly one and w_n is zero. Note in particular that

$$(w_o - w_1) + (w_1 - w_2) + (w_2 - w_3) + \ldots + (w_{n-1} - w_n) = 1 \qquad \ldots(6)$$

To illustrate the stationary case with its balanced stock assume that the asset has a life of 3 years. The book value of a balanced stock will then be

$$B_t = w_2 p_o + w_1 p_o + w_o p_o \qquad \ldots(7)$$

This is clearly independent of t and a function of n only. The annual depreciation charge is

$$D_t = (w_o p_o - w_1 p_o) + (w_1 p_o - w_2 p_o) + (w_2 p_o - w_3 p_o)$$

$$= p_o \qquad \ldots(8)$$

which is just sufficient to replace the asset that expires. Note especially that this is a constant <u>and</u> that it is independent of the allocation method used. The annual revenue R_t is clearly a constant 3q in this case or more generally nq. It follows therefore that the <u>net income</u> of the firm, namely

$$N_t = (3q - p_o) \qquad \ldots(9)$$

will be constant and have a value completely independent of the allocation method in use. It follows that the accounting rate of return will be

$$r_t = \frac{R_t - D_t}{B_t}$$

$$= \frac{nq - p_o}{p_o \sum_{s=0}^{n-1} w_s} \qquad \ldots(10)$$

It follows further that if r_t is to equal the market rate we must have

$$nq - p_o = \rho\, p_o \sum_{s=0}^{n-1} w_s \qquad \ldots(11)$$

But
$$nq - p_o = q\left\{n - \sum_{s=1}^{n} v^s\right\} \qquad \ldots(12)$$

It follows that

$$q(1 - v)\sum_{s=0}^{n-1}(n-s)v^s = \rho\, p_o \sum_{s=0}^{n-1} w_s$$

or that
$$\sum_{s=0}^{n-1} w_s = \frac{qv}{p_o}\sum_{s=0}^{n-1}(n - s)v^s \qquad \ldots(13)$$

This is the necessary and sufficient condition for the accounting rate of return to equal the internal rate of return. A sufficient, but not necessary condition is to use economic depreciation i.e.,

set
$$w_t = \frac{q}{p_o}\sum_{s=1}^{n-t} v^s \qquad \ldots(14)$$

since simple substitution shows that

$$\sum_{t=0}^{n-1} w_t = \frac{q}{p_o}\sum_{t=0}^{n-1}\sum_{s=1}^{n-t} v^s = \frac{qv}{p_o}\sum_{t=0}^{n-1}(n - t)v^t \qquad \ldots(15)$$

Formally therefore we may write

<u>Proposition I:</u> A necessary and sufficient condition for the accounting rate of return to equal the internal rate in a stationary state with constant quasi-rents is for the allocation scheme defined by the weights $\left\{w_s\right\}_{s=0}^{n}$ to be such that

$$\sum_{s=0}^{n} w_s = \frac{qv}{p_o}\sum_{s=0}^{n-1}(n - s)v^s \qquad \ldots(16)$$

Corollary: A sufficient condition for this equality to hold is that

$$w_t = \frac{q}{p_o} \sum_{s=1}^{n-t} v^s \qquad \ldots(17)$$

The main purpose of this first proposition was to develop some notation and in particular place a rather well known result in a more general format. It is clear from the result that in general there will not be an equality in the stationary state i.e., if the equilibrium condition defined in the previous section be a stationary equilibrium, we should not expect equality of accounting rates, and internal rates of return. It is of some interest then to track the potential discrepancy in specific cases. This has been done by numerous authors, in particular Harcourt (1965). Two results are reported here merely as illustrations. The first assumes straight line depreciation when

$$w_t = (1 - t/n) \qquad \ldots(18)$$

so that

$$B_t = \frac{p_o \; (n + 1)}{2} \qquad \ldots(19)$$

and

$$r_t = \frac{2 \; (q - p_o)}{(n + 1) p_o}$$

$$= \frac{2}{(n + 1)} \left\{ \frac{n \rho}{1 - v^n} - 1 \right\} \qquad \ldots(20)$$

Clearly r_t is a function of n and

$$r_t = \rho \quad \text{when } n = 1$$

but

$$r_t \rightarrow 2\rho \quad \text{as} \quad n \rightarrow \infty$$

A simulation of the relationship reveals that the relationship is monotonic and that the divergences are not dramatic given the range of values one is likely to encounter. It does however suggest that a certain amount of care must be exercised in making inter-industry comparisons where the length of the asset lives are very different. One example might be the case where intangible assets such as advertising are expensed and therefore have an effective life of one year, in comparison with a steel plant, say. A few excerpts from the simulation are shown below to give the flavor of the divergences.

Table 1

n	Simulation - straight line depreciation				
1	.0600	.0800	.1000	.1200	.1400
2	.0606	.0810	.1016	.1223	.1431
5	.0623	.0841	.1063	.1290	.1521
10	.0652	.0891	.1140	.1400	.1668
20	.0708	.0988	.1285	.1598	.1924
50	.0852	.1211	.1586	.1969	.2357
100	.0994	.1387	.1782	.2178	.2574

If we assume on the other hand that accelerated depreciation, say sum of year digits, is being used then where

$$d = n(n + 1)/2 \qquad \text{...(21)}$$

we have in the stationary state with a balanced stock

$$w_s = 1 - \frac{2sn - s(s - 1)}{2d} \qquad \text{...(22)}$$

and

$$\sum_{s=0}^{n-1} w_s = \frac{n + 2}{3} \qquad \text{...(23)}$$

when
$$r_t = \frac{nq - p_o}{p_o \sum_{s=0}^{n-1} w_s}$$

$$= \frac{3(nq - p_o)}{p_o(n + 2)}$$

$$= \frac{3}{(n + 2)} \left\{ \frac{n}{1 - v^n} - 1 \right\} \qquad \ldots(24)$$

Once again

$r_t = \rho$ when $n = 1$

but now $r_t \to 3\rho$ when $n \to \infty$

Simulations reveal that the divergence is fairly small for the types of parameters one is likely to encounter in practice as before. A selection of the results are shown below.

Table 2

n	Simulation - sum of year's digits depreciation				
1	.0600	.0800	.1000	.1200	.1400
2	.0682	.0912	.1143	.1376	.1609
5	.0814	.1081	.1367	.1659	.1956
10	.0897	.1226	.1569	.1925	.2293
20	.1014	.1414	.1840	.2289	.2754
50	.1253	.1781	.2333	.2897	.3467
100	.1476	.2060	.2647	.3235	.3824

With these results in hand we can now turn to the case of the firm and, by assumption, the economy that is growing at a steady rate (g-1) i.e., we turn our attention to the relationship between the accounting rate of

return and the internal rate of return in a steady state growth equilibrium. The following proposition is both extremely general and quite startling in its simplicity.[2]

Proposition II: In a steady state growth equilibrium where the rate of growth (g-1) is equal to the internal rate of return , the accounting rate of return will always equal the internal rate of return regardless of the depreciation method used or the pattern of quasi-rents generated by the assets provided the firm hold a balanced stock of assets. Symbolically

$$g = 1 + \rho \rightarrow r = \rho$$

Proof: Assume, by way of example, that the asset has a life of three years and that the firm starts its life with a single asset that costs p_o. Then labelling assets a, b, c,... one has the book values

	Start Yr.1	Start Yr.2	Start Yr.3	Start Yr.4	Start Yr.5
a	p_o	w_1p_o	w_2p_o		
b		p_og	w_1p_og	w_2p_og	
c			p_og^2	$w_1p_og^2$	$w_2p_og^2$
d				p_og^3	$w_1p_og^3$
e					p_og^4

Clearly $B_4 = p_og^4 \ (g^{-1} + w_1g^{-2} + w_2g^{-3})$

and in general

$$B_t = p_og^t \sum_{j=0}^{n-1} w_jg^{-j-1} \qquad \ldots(25)$$

Annual depreciation will be reported at the end of the year under consideration

and in general will be

$$D_t = p_o g^t \sum_{j=0}^{n-1} (w_j - w_{j+1})g^{-j-1} \qquad \ldots(26)$$

We now allow the asset p_o to generate a completely general quasi-rent pattern $\{q_s\}_{s=1}^{n}$ when

$$R_t = g^t \sum_{j=1}^{n} q_j g^{-j} \qquad \ldots(27)$$

But

$$r_t = \frac{R_t - D_t}{B_t}$$

$$= \frac{g^t \sum_{j=1}^{n} q_j g^{-j} - p_o g^t \sum_{j=0}^{n-1} (w_j - w_{j+1})g^{-j-1}}{pg^t \ {}_{j=0}^{n-1} w_j g^{-j-1}}$$

$$= \frac{{}_{j=1}^{n} q_j g^{-j} - p_o \ {}_{j=0}^{n-1} (w_j - w_j +1)g^{-j-1}}{p_o \ {}_{j=0}^{n-1} w_j g^{-j-1}} \qquad \ldots(28)$$

Now assume that $g = (1 + \rho) = v^{-1}$

and note that $\sum_{j=1}^{n} q_j v^j = p_o$

when $$r_t = \frac{1 - \sum_{j=0}^{n-1} w_j v^{j+1} + \sum_{j=0}^{n-1} w_{j+1} v^{j+1}}{{}_{j=0}^{n-1} w_j v^{j+1}} \qquad \ldots(29)$$

However $$\sum_{j=0}^{n-1} w_j v^{j+1} = v \sum_{j=0}^{n-1} w_j v^j$$

and $$\sum_{j=0}^{n-1} w_{j+1} v^{j+1} = \sum_{j=1}^{n} w_j v^j$$

and since $w_n = 0$ and $w_o = 1$

$$1 + \sum_{j=1}^{n} w_j v^j = w_o v^o + \sum_{j=1}^{n-1} w_j v^j + w_n v^n$$

$$= \sum_{j=0}^{n-1} w_j v^j$$

$$\Rightarrow \qquad r_t = \frac{-v\sum_{j=0}^{n-1} w_j v^j + \sum_{j=0}^{n-1} w_j v^j}{v\sum_{j=0}^{n-1} w_j v^j}$$

$$= \frac{1 - v}{v}$$

$$= \rho \qquad \text{Q.E.D.}$$

There are no restrictions placed on the quasi-rent pattern whatsoever. Moreover the <u>only</u> restrictions placed on the book value is that $w_o = 1$ and that $w_n = 0$ i.e., that the assets should be entered at cost and that it should be written off completely at the end of its useful life. Neither of these conditions deserve to be called restrictions. Apart from them, the weights used can be as much <u>above</u> 1, implying capital appreciation, or as much below 1, implying capital depreciation, as one chooses, provided of course the weights are used consistently.

One question that must remain in the readers' minds is the generality of the condition that the growth rate be the rate of profit. The following proposition drawn from modern capital theory and in no sense original suggests that the result is considerably more than an oddity.

<u>Proposition III</u>: (The Golden Rule) In a steady state economy with growth rate p, a maximum consumption optimum is achieved when the growth rate is equal to the rate of profit.

$$\text{i.e., } c^* = \max c \quad \Leftarrow \quad p = \rho$$

Proof: Assume that the technology of the economy may be defined by a constant returns to scale neo-classical production function

$$Q = F(K, L) \qquad \text{...(30)}$$

where K is the flow of capital, and L the flow of labor. Assume further that labor grows at an exogenous rate g so that

$$\dot{L} = pL \qquad \text{...(31)}$$

Assume further that the economy saves, or equally invests, at a rate such that

$$\dot{K} = sQ \qquad \text{...(32)}$$

We may now write

$$Q = LF(K/L, 1)$$

or

$$q = f(k) \qquad \text{...(33)}$$

with the usual assumptions that

$$f(0) = 0 \qquad f'(k) \to \infty \quad \text{as } k \to 0$$

$$f'(k) > 0 \qquad f'(k) \to 0 \quad \text{as } k \to \infty$$

$$f''(k) < 0$$

It is easy to demonstrate that

$$\dot{k} = \left(\frac{\dot{K}}{K} - \frac{\dot{L}}{L}\right)\frac{K}{L} \qquad \text{...(34)}$$

or

$$\dot{k} = sf(k) - pk \qquad \text{...(35)}$$

The characteristics of this differential equation, which defines the growth path of the economy, have been well studied. It is known for instance that the economy will tend to a value k* such that

$$sf(k^*) - pk^* = 0$$

and that this value is globally stable. (For details see Burmeister &

Dobell, 1970). A phase diagram of the equation appears as follows

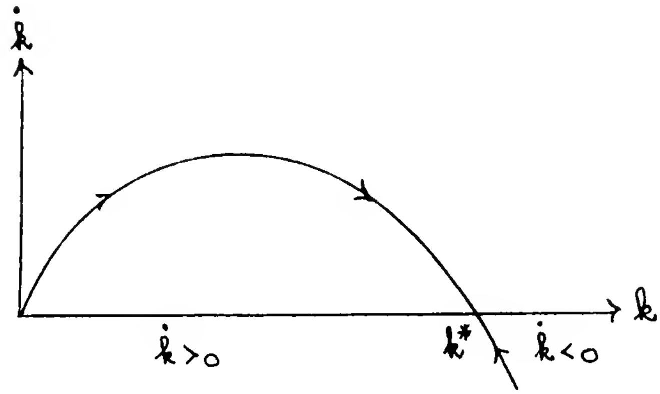

Fig. 2.1

The equation is also easily interpreted since sQ represents the total investment of the economy, and pK is the investment flow necessary to have capital grow at the same rate as labor - commonly refered to in the literature as capital widening. The difference then of sQ - pK is the surplus available to equip the labor force with more capital per capita i.e., to undertake capital deepening. The final equilibrium position is achieved when capital deepening ceases. But clearly at that point by (34)

$$\frac{\dot{K}}{K} - \frac{\dot{L}}{L} = 0$$

i.e.,

$$\dot{K} = pK \qquad \ldots(36)$$

In words, capital at that point will increase at the same rate as the growth in the labor force. In terms of our earlier notation in equilibrium

$$(g - 1) = p$$

Now at this point consumption per capita is defined by

$$c = (1 - s)q^* = (1 - s)f[k^*(s)]$$
$$= f(k^*) - pk^* \qquad \ldots(37)$$

It follows that

$$\frac{dc}{ds} = f'(k^*) - p \frac{dk^*}{ds}$$

where $f'(k) = \frac{df}{dk}$

This last is equal to zero and a maximum when

$$f'(k^*) = p$$

But

$$f'(k) = F_K(K, L)$$

$= \text{marginal product of capital}$

$= \text{rate of profit by definition}$

$= \rho$ Q.E.D.

i.e., a consumptive optimal is achieved when the growth of capital, equals the growth of the labor force, equals the rate of profit. In the notation of Proposition II, the optimal growth path for the economy, as here defined, is achieved when

$$g = 1 + \rho$$

In other words, there is a very real sense in which the results of the previous proposition was not a curiosum. Given the assumption that the economy will tend to an optimal situation there is a strong presumption that there will always be a tendency for the accounting rate of return to equal the internal rate of return.

To emphasize the strength of the results that have been obtained the following corollaries, are offered.

Corollary 1: In a steady state growth equilibrium, using different methods of accounting for different assets, in particular expensing as against capitalizing, will have no effect on the accounting rate of return.

Corollary 2: In a steady state growth equilibrium the existence of assets with different lives will have no effect on the accounting rate of return.

Corollary 3: In a steady state growth equilibrium allocating the cost of the asset over a period that is not equal to the life of the asset will have no effect on the accounting rate of return. (Allocating over a period less than the life is obvious. Allocating over a period greater than the life is done by simply extending the "life" of the asset with zero quasi-rents.)

Corollary 4: In a steady state growth equilibrium if the growth rate vary but in such a manner that it be perfectly correlated with variations in the internal rate of return, then the accounting rate of return will always equal the internal rate of return.

All the propositions have treated revenue as cash revenue. However, one of the most fundamental features of accounting is the accrual concept. Revenue is recognized at the point of sale, not at the moment of the receipt of cash. Expenses are matched with benefits and are not equated with expenditures. It is simple if tedious to show that if the revenue and expenses in a period are the properly time discounted values of cash receipts and expenditures all the above results flow through without change i.e., we have,

Corollary 5: In a steady state growth equilibrium accrual accounting will not affect the rate of return.

It must be borne in mind, of course, that all these results have been equilibrium results. What should be clear from all the above is that most of our accounting problems derive from a state of disequilibrium. One question that, therefore, requires addressing is the relevance of equilibrium results to accounting practice as theory. But the identical question can be addressed to the economist who has made such a lot of use of the equilibrium method. The English economist G. L. S. Shackle (1961) gave this response:

> "When we have no theory about economic affairs, no state of those affairs and no temporal succession of states seems inconceivable. A theory restricts the conceivable states and successions of states to those in which the relations between quantifiable things in the economy conform to some specified rules....This meaning of 'economic theory' leaves unlimited the number of different theory-classifying schemes we can set up....One dichotomy is between equilibrium and development theories. Equilibrium is a test that selects for the economist one particular situation out of an infinity of situations and justifies his calling attention to it as something special. Judged by the smallness of the ratio of what it accepts to what it rejects, no other test seems able to rival in its selective power. No other test, it may also be claimed, can state so sharply in what the accepted differs from the rejected situations....On the most general grounds, equilibrium has great claims as an economiser of thought. To dispense with it has meant, in practice, to be reduced to mere factual enumeration. For ninety years few economists, save the German historical school, have based their theories upon any other principle. Even those most anxious to disparage it as a description of what is and, still more, of what ought to be, have nonetheless needed it as a means of understanding and of accounting for what is."

Now, it would be foolish indeed, to suggest that the economy is in a state of longrun equilibrium, or that all firms are growing at the same rate as each other and, moreover, at the same rate as the population or work force. What one wants to suggest instead is that, if our assumptions hold, what will be the tendency of the economy and, more particularly, what will be the tendency of the accounting system. The answer, as we have seen is that the accounting system tends to behave very well -

Assume further that $g = (1+\delta)(1+\rho) = \alpha^{-1}v^{-1}$

when

$$r_t = \frac{\sum_{j=1}^{n} q_j v^j (\alpha^j - 1)}{p_o \sum_{j=0}^{n} w_j \alpha^{j+1} v^{j+1}} + (g-1) \qquad \ldots(39)$$

and note $r_t = g - 1$ when $\alpha = 1$ i.e., $\delta = 0 \Rightarrow g = 1 + \rho$

$r_t > g - 1$ when $\alpha > 1$ i.e., $\delta < 0 \Rightarrow g < 1 + \rho$

$r_t < g - 1$ when $\alpha < 1$ i.e., $\delta > 0 \Rightarrow g > 1 + \rho$

Diagrammatically this implies that if we plot r_t against $G = (g - 1)$, and draw a 45° ray from the origin that accounting rates will fall above or below the ray depending on the relationship of G to ρ. Furthermore we know by Proposition II and sequitur that all accounting rates converge to ρ when G converges to ρ. By (39) this convergence for any given accounting method is clearly monotone depending as it does only on the α tending to one. Pulling this together in a diagram we have for any given accounting method a locus that will look approximately like this - approximately because the locus is not a straight line.

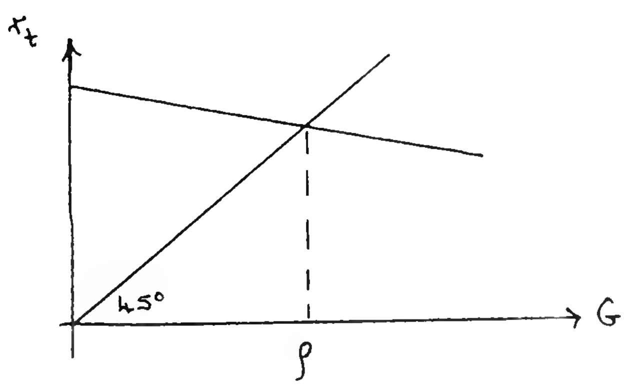

Figure 1

But G = 0 is simply the stationary result as outlined in Proposition I. The results tabulated in Table I give us a series of points for straight line depreciation with assets of different lives. These can be plotted on the vertical axis. We know from Proposition II that all these loci will converge to a point at $G = \rho$. From Proposition IV we know that all thes loci converge monotonically. Connecting them all up we have something like this.

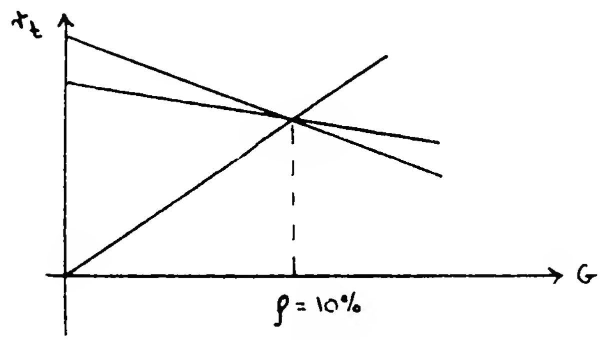

Figure 2

Note incidentally that this gives a quick and dirty way to arrive at accounting rates for any growth rate - simply calculate the stationary rate and connect it to the internal rate. It will, of course, only be approximate. Note also that (39) is a generalization of (10) i.e., the substitution of G = 0 into (39) yields (10) as, of course, it should. And equally obviously we can draw diagrams for any other combination such as straight line depreciation versus sum of years depreciation for a fixed asset life, say 10 years.

Note particularly how the various alternatives maintain their ranks over the whole range of values of $G < \rho$ and then immediately reverse their ranks as they pass to the range $G > \rho$. If the firm be close to a Golden rule growth path and growth rates switch from below the consumption maximum to above it, or *vice-versa*, relative accounting rates of return would switch.

However, <u>great</u> care must be taken in interpreting these diagrams. The appearance of monotonicity is <u>only</u> an appearance. Firms do <u>not</u> move sideways across the page. Missing from the diagram - as from most economic diagrams incidentally - is the time domain which is best thought of as emerging vertically from the page. In the imagery of Robinson (1953-4) the loci in the above diagrams represent accounting rates on a series of islands. Each island is in a longrun steady state equilibrium with a growth rate G. It is <u>we</u> who are moving from island to island, <u>not</u> the growth rates on the islands changing. As <u>we</u> move from island to island so we shall see a wide (small) dispersion of accounting rates on islands where the growth rate is (not) significantly different from the internal rate. And as <u>we</u> move from islands where growth rates are significantly different to islands where the difference is insignificant <u>we</u> shall see a steady convergence of accounting rates. <u>But</u> for any one island to become similar to any other island that island's accounting rates will go through a period of adjustment characterized in many cases by a damped oscillatory convergence. Diagrammatically we have potentially something like this.

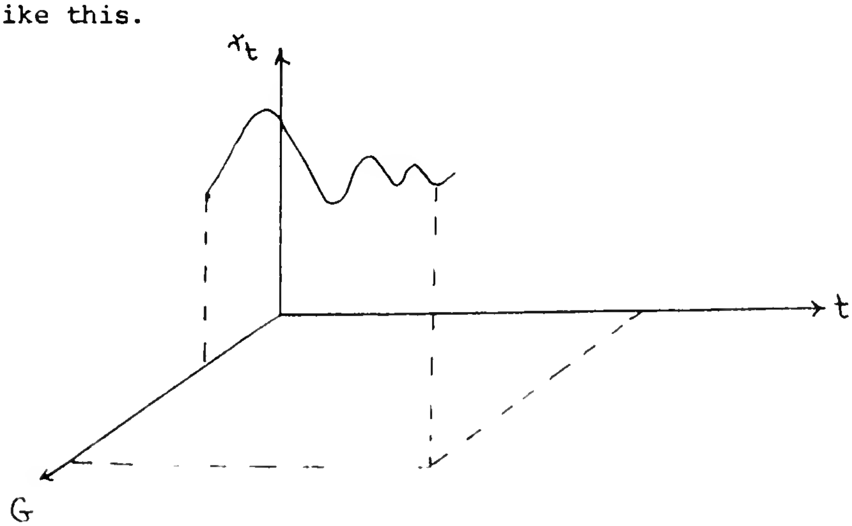

Figure 3

This result was developed in a simulation study by Livingstone and Salamon (1970) and, subsequently, extended by Livingstone and Van Breda (1976). The results have been extended still further in Van Breda (1978).

Conclusions: Many, if not all, of the results in this paper have been known to either accountants or economists. What this paper has done is to draw them together to demonstrate that what was often treated simply as an oddity is in actual fact an equilibrium result. Two basic conclusions emerge. One is that the accounting system responds extremely well in equilibrium. Net income is independent of arbitrary allocations.[3] The accounting rate of return reflects the economic or internal rate of return.

The other basic conclusion is that almost all our difficulties arise from being out of an equilibrium situation. Many of our cost allocation procedures could be likened to an economist trying to determine prices when supply does not equal demand - in a word impossible. However, what we can do is to trace the behavior of the accounting system around the equilibrium point and its behavior as the firm moves towards equilibrium. A few steps towards this were taken in this paper.

Given that the equilibrium method has proved as powerful in other disciplines the question must arise whether we accountants could not make more use of it. This paper is a first attempt to answer that question.

FOOTNOTES

1. Stauffer (1971) in a very pretty article in the Bell Journal derives all these results, and more, for the continuous case.

2. As with the first proposition, the result is not new. A similar result may be found in Solomon (1956) for the case of straight line depreciation and for the general, continuous case in Stauffer (1971).

3. The existence of arbitrary allocations has been dealt with at length by Thomas (1974).

BIBLIOGRAPHY

Bailey, M., "Formal Criteria for Investment Decisions," Journal of Political Economy, 67:476-88, 1959.

Bierman, H. and Smidt, S., The Capital Budgeting Decision, 4th ed., MacMillan, 1975.

Burmeister, E. and Dobell, A. R., Mathematical Theories of Economic Growth, MacMillan, 1970.

Debreu, G., Theory of Value, Yale University, 1959.

Fisher, I., The Rate of Interest, MacMillan, 1907.

Fisher, I., The Theory of Interest, MacMillan, 1930.

Harcourt, G. C., "The Accountant in a Golden Age," Oxford Economic Papers, 17:66-80, 1965.

Hirshleifer, J., Investment, Interest, and Capital, Prentice-Hall, 1970.

Horngren, C. T., Cost Accounting: A Managerial Emphasis, 4th edition, Prentice-Hall, 1977.

Livingstone, J. L. and Salamon, G. L., "Relationship between the Accounting and the Internal Rate of Return Measures: A Synthesis and an Analysis," Journal of Accounting Research, 8:199-216, 1970.

Miller, M. H., and Modigliani, F., "The Cost of Capital, Corporation Finance and the Theory of Investment," American Economic Review, 48:261-297, June 1958.

Robinson, J., "The Production Function and the Theory of Capital," Review of Economic Studies, 21:81-106, 1953-4.

Sarnat, M. and Levy, H., "The Relationship of Rules of Thumb to the Internal Rate of Return: A Restatement and Generalization," Journal of Finance, 24:479-490, 1969.

Shackle, G. L. S., Expectation, Enterprise and Profit, Aldine, 1970.

Shackle, G. L. S., "Recent Theories Concerning the Nature and Role of Interest," Economic Journal, 71:209-54, 1961.

Shillinglaw, G., Managerial Cost Accounting, 4th edition, Irwin, 1977.

Solomon, E., "Arithmetic of Capital Budgeting Decisions," Journal of Business, 29:124-29, 1956.

Solomon, E., "Return on Investment: the Relation of Book-Yield to True Yield," Research in Accounting, R. K. Jaedicke, Y. Ijiri and O. Nielsen, editors, AAA, 1966.

Stauffer, T. A., "The Measurement of Corporate Rates of Return: A Generalized Formulation," Bell Journal of Economics and Management Science, 2:434-469, 1971.

Swalm, R. O., "On Calculating the Rate of Return on Investment," The Journal of Industrial Engineering, 9:99-103, 1958.

Thomas, A. L., The Allocation Problem: Part Two, American Accounting Association, Studies in Accounting Research, No. 9, 1974.

Van Breda, M. F., "Accounting for the Traverse," Unpublished Working Paper, Sloan School, MIT, 1978.

Van Breda, M. F., "Capital Budgeting using Terminal Amounts," Sloan School, MIT, WP 952-77, 1977.

Van Breda, M. F. and Livingstone, J. L., "The Relationship Between Accounting and the Internal Rate of Return Measures: A Reply," Journal of Accounting, Vol. 14, No. 1, Spring 1976.

Van Horne, J. C., Financial Management and Policy, 4th ed., Prentice-Hall, 1977.

SOME PERSPECTIVES ON COMPUTERIZED
MANAGEMENT DECISION MAKING SYSTEMS*

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CISR-33

Sloan WP 983-78

March 1978

*Presented at the IEEE "COMPSAC 77" Conference, Chicago, Illionis -- November 1977

SOME PERSPECTIVES ON COMPUTERIZED MANAGEMENT DECISION MAKING SYSTEMS

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The title of this paper is somewhat of a misnomer, perhaps a more appropriate one would be "Some Perspectives on Computerized Management Decision Support Systems." I personally think that the difference between decision making and decision support is a key concept. The use of computers to support decision making and the use of computers to actually make decisions are both important areas, particularly in a management setting the way it is in the 1970's. The technological shifts over the past two years have made new things possible, and the range of decisions that can now be usefully supported by computers is vastly different than it was even three or four years ago. There are a number of technological changes that have made this possible, but in particular three come to mind: the first of these is the development of powerful, robust, low cost mini-computers, the second is the availability of good interactive terminals, and the third the availability of decent data management languages which permit accessing data in a variety of ways at problem solution time. Each of these areas, and others, can and will have further development, but we have now reached the stage where we can deliver a computer "system" that contains enough power to be used on problems of real world complexity. In fact we have reached the stage where Decision Support Systems (DSS's) are not just imaginary toys of management but actually exist and are used as

very effective tools by line and staff management.[1,5,8]

To provide a little perspective on the status and differences between these two broad areas let us look at some examples of each. The framework that is used to help structure the management setting is provided by a simple matrix. This is developed elsewhere[7] but consists for our purposes here of the two axes in Figure 1. For one of these we are using Anthony's familiar Strategic Planning Management Control Operations Control view of the classes of decision that exist in an organization. The other axis consists merely of four of the functional areas in a firm. Using this structure to think about the status of computer based decision making systems in organizations today it is possible to see that there are very few such systems actually in use in organizations in the Strategic Planning and Management Control areas. It is not that computers are never used in these domains, but simply that they form a trivial part of the eventual decision process. However, the picture is very different in the operational control aspects of organizations. In all the functional areas there is almost always an example in any given firm, of the computer being used to actually make decisions. Taking each of these functional areas in turn we have the following examples.

Marketing - Technical Specification.

In technical areas some firms find that detailed specifications can be effectively provided by computer. The salesman inputs the customer requirements for the system and the computations are then made that provide the detail requirements, costs and prices.

Production - Refinery Scheduling.

Examples in the production area are by now legion. Real time process control for example is a commonplace application in process industries and has advanced enormously in the last 15 years. One only has to visit a modern refinery to realize the total dependence such a complex production system has on the process control and computer based decisions that are made throughout. It is in fact a classic illustration of computer based decision making.

Production-Line Balancing.

There are many examples in non-process applications, they are typically not real time applications, but they exploit our advances in math programming to provide optimal decisions in some aspect of the manufacturing process. For those companies with production lines manufacturing relatively standard products, and where the production lines must be balanced so as to provide maximum through-put from the factory we find that computer based programming systems are making decisions that used to be made by managers and first line supervision.

Finance - Credit Checks.

For a number of companies the question of whether credit should be extended to a customer for a particular purchase is a matter of some importance. Where the company is large with a fast moving business, computer based systems are employed which have the necessary decision rules to make a decision as to whether to extend credit or not.

Personnel - Availability.

By maintaining a skills and preference data base some firms make personnel assignments on the basis of computer executed decision rules. The most extreme example of this is probably the United States Military forces, but such systems are also used in private industry.

These examples are obviously merely illustrative. However in these and all other decision making systems the problem involved has a number of common characteristics. For example;

1. They are structured problems. That is using Simon's[7] terminology the three problem solving stages of intelligence, design, and choice can be specified unambiguously ahead of time. The data for each of these stages are known in advance and the relationships and decision rules that apply are also known.

2. These problems exist in a predictable environment or if that does not hold in an environment where there is not time pressure.

3. There is no special requirement for communication between various managers or between sections of the organization.

Where problems have characteristics such as these then computers can be used to replace or supplant the human decision maker and one can build a decision making system which is effective and valuable to the organization. It is an interesting and important area and will continue to be of major significance to companies and their data processing and operations research groups. However, as we suggested above, the recent changes in technology have made it

possible to develop systems in a different but related area which we call Decision Support Systems. The emphasis here as the name suggests is to support managers in making complex decisions and not to focus on replacing them. Such an approach is appropriate for semi-structured decisions, that is decisions where there exists a sufficient degree of ambiguity that it cannot be automated and given to the machine in its entirety and yet the situation is also such where pure management intuition can be improved upon and therefore the human being alone is not doing as good a job as is possible. In such a situation where neither the human nor the machine do as good a job as the two combined we have found an extraordinary mushrooming of applications over the last two years. Using the same framework to look at a company as we have done in the case of fully structured problems we have the situation as depicted in Figure 2. Across all three levels of management decision making we find actual systems in use in companies today. Some sample examples are given in Figure 2 and the numbers in parantheses with each of the examples refers to the items in the bibliography which provide a fuller discussion of the application. These numbered items are not discussed here but can be referred to in the original source documents. However in Figure 2 there are two areas in which there does not exist adequate written material at this point and so a word or two is in order.

In the area of Finance and Strategic Planning a number of firms have had models to help look at the impact of potential acquisitions on the financial status of the acquiring firm. The details and use of such systems are obviously closely held by the companies in question as there is the inevitable confidential nature of the

material with which they are dealing. However firms such as Westinghouse claim to find these interactive tools to be of inestimable value and from our research projects here at M.I.T. they appear to be in very wide use among the major corporations. The second area without a reference is in the Strategic Planning-Personnel domain where the illustrative example has to do with long term manning needs. Both the military and civilian government agencies have had extensive experience with computer based support for this area. NASA has a number of internal documents which provide some insight into the use of interactive planning models to support key decisions on the manning levels and shifts in these as NASA looked out over its somewhat uncertain future.

There is obviously not enough space to describe any one of these in any detail nor have we begun to cover the range of functions in a firm, or the range of types of DSS's that actually exist at this point in time. The Center for Information Systems Research (CISR) at the Sloan School of Management at M.I.T. has a number of fascinating case studies documenting aspects of this area and from this work it is clear that there is an enormous range of DSS's that are possible. Some of these run from very small time-sharing based systems with a budget of a few thousand dollars and others run to major projects involving substantial dollars and substantial organizational effort. For example the very innovative moves recently by International Harvester in providing interactive support for their purchasing agents, who are responsible for purchasing well over 4 billion dollars annually,

shows a case with high payoff and also demonstrates how some relatively rigid tools such as IBM's IMS can be exploited to provide effective information to managers if the focus on decision support is made clear.

In all of these DSS examples we can say, using Simon's view of decision making, that either one or two of the "Intelligence", "Design" or "Choice" stages in the decision making process continue to require management judgment. As a result the overall decision process cannot be said to be structured, and, therefore, requires the manager to stay actively involved in the decision making process. Thus we find in all of the applications listed above that the DSS's developed are management tools used by managers either directly themselves or by their personal staff. Paranthetically it is not a coincidence that all of these example systems have not involved data processing departments to date.[8] The reasons for this lack of involvement are several, among which are the phenomena that the process of building such systems, the models that are appropriate to help support managers in these conditions, the kinds of computers, and the type of analyst that is successful in these efforts, are all quite different from the classical MIS applications. This point is not elaborated on here but is developed further elsewhere.[8]

To illustrate the differences between decision making systems and decision support systems, and at the same time to highlight some of the many similarities between the two let us look at an illustration using a set of games.

Tic-tac-toe is a fully structured game in which we have long been able to build a computer system that plays a faultless game. It will always win or draw. This degree of structure is somewhat deceptive as can be seen when one looks at the five year old child who plays the game. The child may well understand the surface decision rules so that in fact the child can play the game, on the other hand the child does not understand the deep structure involved (borrowing the term from Minsky) and therefore is unable to play a faultless game.

Checkers on the other hand is not as easy a game. We understand surface rules and we can write computer programs to play a very respectable game of checkers. However it is too large to allow brute force answers and so a number of people have been working on the deep structure which is yielding to their efforts slowly. In fact we have been at the point for some years where the heuristics from good checkers players when programmed as part of a checkers playing system can be very effective. As a result of this work with heuristics there are now a number of excellent checkers playing systems that do very well and in fact can regularly beat a decent player. Samuels work at IBM is perhaps one of the earliest examples of this.

Chess presents a different picture. It is effectively unstructured still and there does not seem to be any evidence that the middle game has yielded thus far to any of the research efforts going on around the world. In fact we are still at the point that no game playing computer system provides any serious opposition to a good chess player. Now supposing we decided to change the

ground rules in working with computers and chess. Instead of struggling to build a computer program that would replace the human chess player we take as our goal a quite different one, not commonly used in the past, that is to provide support to the chess player so as to improve the player's game.

If we were to do this we might find ourselves in the following situation: the computer would be dominant in two areas. In the opening game the computer could draw on a data base of past "great moves" and provide the human player with a substantially wider basis of experience. Similarly in the end-game it would be possible to turn the play over to the computer system and allow it to use algorithms that involve exhaustive search. Thus the computer would likely dominate the human it is supporting in both the "opening game" and the "end game" although in quite different ways in both cases. However in the "mid game" where computers have been singularly inept in the work thus far, the computer would drop to a supportive role and would merely suggest obvious moves or pitfalls if it sees them, and otherwise react in a "what if" mode to suggested moves by the human player.

With such a support system it seems highly likely that the chess game of many players would be substantially improved.

In an analogous fashion it is true that the performance of companies and of managers can be improved if computers are used to provide the classical decision making activities as well as providing decision support activities. By shifting the ground rules on the way computers are used in companies to include not only replacing managers and clerical work forces but also to include the notion of supporting them we add a whole new range of activities to the things

that can be done in corporations and from the experience gained thus far these are activities with substantial pay off. To many people coming from a background of elegant algorithms and extraordinarily complex computer systems this movement into decision support systems seems somewhat trivial and not particularly challenging. It does not require the phenomenal sense of completeness that a human replacing system requires, simply because the manager or clerk is still there and can still provide the necessary guidance and insight to override the system it is seems appropriate. From talking to the builders of such DSS's thus far it appears that in fact this is an area that is no less challenging either conceptually or in practice than the previous computer work.[8,11] The technology available to managers and computer people has changed dramatically over the last two years, this change has opened up a whole new domain that contains considerable potential. I would expect that over the next few years we are going to see a lot of exciting and quite different developments than those we have seen over the previous ten. It is likely to be an interesting challenge.

Figure 1

Examples of Computer-Based Decision Making

	Strategic Planning	Management Control	Operational Control
Marketing			Technical specification
Product	Very few Computer-Based Decision Making Systems in these Areas.		Refinery scheduling Production Line Balancing
Finance			Credit Checks
Personnel			Personnel availability

Figure 2

Examples of Computer-Based Decision Support Systems

	Strategic Planning	Management Control	Operational Control
Marketing	New Product Introduction (14)	Brand Management (9)	Media Selection (10)
Production	New Facilities Location (4)	Setting Production Plan (13)	Portfolio Management (6)
Finance	Impact of Acquisition	Setting Budget Levels (11)	Cash Flow Management (12)
Personnel	Long-Term Manning Needs	Staffing Strategies (3)	Personnel Selection (3)

REFERENCES

1. Alter, Steven L., "A Study of Computer Aided Decision Making in Organizations", Ph.D. Thesis, Sloan School of Management, Massachusetts Institute of Technology, 1975.

2. Anthony, Robert N., Planning and Control Systems: A Framework for Analysis, Division of Research, Graduate School of Business Administration, Harvard University, Boston, 1965.

3. Berger, Paul and Franz Edelman, "IRIS: A Transaction-based DSS for Human Resource Management", Data Base, Vol. 8, No. 3, Winter 1977.

4. Buffa, E.S., G. C. Armour and T. E. Vollman, "Allocating Facilities with CRAFT", Harvard Business Review, Vol. 42, No. 2, March-April 1964.

5. Carlson, Eric D., Editor, "Proceedings of a Conference on Decision Support Systems", Data Base, Vol. 8, No. 3, Winter 1977.

6. Gerrity, Thomas P. Jr., "Design of Man-Machine Decision Systems: An Application to Portfolio Management", Sloan Management Review, Massachusetts Institute of Technology, Vol. 12, No. 2, Winter 1971.

7. Gorry, Anthony M. and Michael S. Scott Morton, "A Framework for Management Information Systems", Sloan Management Review, Massachusetts Institute of Technology, Vol. 13, No. 1, Fall 1971.

8. Keen, Peter G. W. and Michael S. Scott Morton, Decision Support Systems: An Organizational Perspective, Addison Wesley, Boston, 1978.

9. Little, John D. C., "BRANDAID: A Marketing Mix Model. Part I: Structure; Part II: Implementation, Calibration, and Case Study", Operations Research, Vol. 28, No. 4, July-August 1975.

10. Little, John D. C. and Leonard Lodish, "A Media Planning Calculus", Operations Research, Vol. 17, No. 1, January-February 1969.

11. McCosh, Andrew M. and Michael S. Scott Morton, Management Decision Support Systems, The MacMillan Press, Ltd., London, 1978.

12. McInnes, J. M., "An Organizational Perspective on the Use of Simulation Methods in the Planning and Control Process", Proceedings of the Summer Computer Simulation Conference, Chicago, Illinois, July 1977.

13. Scott Morton, Michael S., Management Decision Systems: Computer-Based Support for Decision Making, Division of Research, Graduate School of Business Administration, Harvard University, Boston, 1971.

14. Urban, Glenn L., "SPRINTER Mod III: A Model for the Analysis of New, Frequently Purchased Consumer-Products", Operations Research, Vol. 18, No. 5, September-October 1970.

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